

$$\mathbf{A} = \{\{\mathbf{a}_0, \mathbf{a}_1\}, \{\mathbf{a}_1, \mathbf{a}_2\}\}$$

$$\mathbf{B} = \{\mathbf{b}_0, \mathbf{b}_1\}$$

$$\mathbf{x}_0 = \{\mathbf{0}, \mathbf{0}\}$$

$$\{\{\mathbf{a}_0, \mathbf{a}_1\}, \{\mathbf{a}_1, \mathbf{a}_2\}\}$$

$$\{\mathbf{b}_0, \mathbf{b}_1\}$$

$$\{\mathbf{0}, \mathbf{0}\}$$

$$\mathbf{r}_0 = \mathbf{B} - \mathbf{A} \cdot \mathbf{x}_0$$

$$\{\mathbf{b}_0, \mathbf{b}_1\}$$

$$\mathbf{p}_0 = \mathbf{r}_0$$

$$\{\mathbf{b}_0, \mathbf{b}_1\}$$

$$\mathbf{p}_0 \cdot (\mathbf{A} \cdot \mathbf{p}_0)$$

$$\mathbf{b}_0 \left(\mathbf{a}_0 \mathbf{b}_0 + \mathbf{a}_1 \mathbf{b}_1 \right) + \mathbf{b}_1 \left(\mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1 \right)$$

$$\mathbf{rsold} = \mathbf{r}_0 \cdot \mathbf{r}_0$$

$$\mathbf{b}_0^2 + \mathbf{b}_1^2$$

$$\mathbf{alpha}_0 = (\mathbf{r}_0 \cdot \mathbf{r}_0) / (\mathbf{p}_0 \cdot (\mathbf{A} \cdot \mathbf{p}_0))$$

$$\frac{\mathbf{b}_0^2 + \mathbf{b}_1^2}{\mathbf{b}_0 \left(\mathbf{a}_0 \mathbf{b}_0 + \mathbf{a}_1 \mathbf{b}_1 \right) + \mathbf{b}_1 \left(\mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1 \right)}$$

$$\mathbf{r}_1 = \mathbf{r}_0 - \alpha_0 * (\mathbf{A} \cdot \mathbf{p}_0)$$

$$\left\{ b_0 - \frac{(a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2)}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)}, \right. \\ \left. b_1 - \frac{(a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2)}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)} \right\}$$

/*sr₁ is short for “simplified r₁”, simplified result of r₁ stored in sr₁

* the following variable names are similar: “s”+“variable name” means the simplified variable.

*/

$$\mathbf{sr}_1 = \mathbf{Simplify}[\mathbf{r}_1]$$

$$\left\{ \frac{b_1 ((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2))}{a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)}, \frac{b_0 ((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2))}{a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)} \right\}$$

$$\mathbf{x}_1 = \mathbf{x}_0 + \alpha_0 * \mathbf{p}_0$$

$$\left\{ \frac{b_0 (b_0^2 + b_1^2)}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)}, \frac{b_1 (b_0^2 + b_1^2)}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)} \right\}$$

$$\mathbf{rsnew} = \mathbf{sr}_1 \cdot \mathbf{sr}_1$$

$$\frac{b_1^2 ((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2))^2}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} + \frac{b_0^2 ((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2))^2}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2}$$

$$\mathbf{srsnew} = \mathbf{Simplify}[\mathbf{rsnew}]$$

$$\frac{(b_0^2 + b_1^2) ((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2))^2}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2}$$

$$\mathbf{beta} = \mathbf{srsnew} / \mathbf{rsold}$$

$$\frac{((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2))^2}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2}$$

In[76]:= **p₁ = sr₁ + beta * p₀**

$$\text{Out[76]= } \left\{ \frac{b_1 \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)}{a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)} + \frac{b_0 \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)^2}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2}, \right. \\ \left. \frac{b_1 \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)^2}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} + \frac{b_0 \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right)}{a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)} \right\}$$

sp₁ = Simplify[p₁]

$$\left\{ \frac{(a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2}, \right. \\ \left. \frac{(a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2) \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right)}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} \right\}$$

/* alpha1 = <r1, r1> / <p1, Ap1> */

alpha₁ = (srsnew) / (sp₁ . (A.sp₁))

$$\left((b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)^2 \right) /$$

$$\left((a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2 \left(\frac{1}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} \right. \right.$$

$$(a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)$$

$$\left(\frac{a_0 (a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} + \right.$$

$$\left. \frac{a_1 (a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2) \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right)}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} \right) +$$

$$\frac{1}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} (a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2)$$

$$\left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right)$$

$$\left(\frac{a_1 (a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} + \right.$$

$$\left. \frac{a_2 (a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2) \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right)}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} \right) \right)$$

salph₁ = Simplify[alpha₁]

$$- \frac{a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)}{(a_1^2 - a_0 a_2) (b_0^2 + b_1^2)}$$

/* r2 = r1 - alpha1*Ap1 */

$$\mathbf{r}_2 = \mathbf{sr}_1 - \mathbf{salpha}_1 * (\mathbf{A} \cdot \mathbf{sp}_1)$$

$$\left\{ \frac{b_1 \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)}{a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)} + \right. \\ \left((a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)) \left((a_0 (a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \right. \right. \\ \left. \left. \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right) \right) \right) / (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2 + \\ \left(a_1 (a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2) \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right) \right) / \\ \left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2 \Bigg) / \left((a_1^2 - a_0 a_2) (b_0^2 + b_1^2) \right), \\ \frac{b_0 \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right)}{a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)} + \left((a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)) \right. \\ \left((a_1 (a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right) \right) / \\ \left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2 + \\ \left(a_2 (a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2) \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right) \right) / \\ \left. \left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2 \right) \Bigg) / \left((a_1^2 - a_0 a_2) (b_0^2 + b_1^2) \right) \Bigg\}$$

In[77]:= **Simplify**[r₂]

Out[77]= {0, 0}

$$\mathbf{x}_2 = \mathbf{x}_1 + \mathbf{salpha}_1 * \mathbf{sp}_1$$

$$\left\{ \frac{b_0 (b_0^2 + b_1^2)}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)} - \right. \\ \frac{(a_1 b_0 + a_2 b_1) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)}{(a_1^2 - a_0 a_2) (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))}, \\ \frac{b_1 (b_0^2 + b_1^2)}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)} - \\ \left. \frac{(a_0 b_0 + a_1 b_1) \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right)}{(a_1^2 - a_0 a_2) (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))} \right\}$$

sx₂ = Factor[x₂]

$$\left\{ \frac{a_2 b_0 - a_1 b_1}{-a_1^2 + a_0 a_2}, \frac{-a_1 b_0 + a_0 b_1}{-a_1^2 + a_0 a_2} \right\}$$

bncg = A.sx₂

$$\left\{ \frac{a_1 (-a_1 b_0 + a_0 b_1)}{-a_1^2 + a_0 a_2} + \frac{a_0 (a_2 b_0 - a_1 b_1)}{-a_1^2 + a_0 a_2}, \frac{a_2 (-a_1 b_0 + a_0 b_1)}{-a_1^2 + a_0 a_2} + \frac{a_1 (a_2 b_0 - a_1 b_1)}{-a_1^2 + a_0 a_2} \right\}$$

Simplify[bncg]

{b₀, b₁}