

$$\mathbf{A} = \{\{\mathbf{a}_0, \mathbf{a}_1\}, \{\mathbf{a}_1, \mathbf{a}_2\}\}$$

$$\mathbf{B} = \{\mathbf{b}_0, \mathbf{b}_1\}$$

$$\mathbf{x}_0 = \{0, 0\}$$

$$\{\{\mathbf{a}_0, \mathbf{a}_1\}, \{\mathbf{a}_1, \mathbf{a}_2\}\}$$

$$\{\mathbf{b}_0, \mathbf{b}_1\}$$

$$\{0, 0\}$$

$$\mathbf{r}_0 = \mathbf{B} - \mathbf{A} \cdot \mathbf{x}_0$$

$$\{\mathbf{b}_0, \mathbf{b}_1\}$$

$$\mathbf{p}_0 = \mathbf{r}_0$$

$$\{\mathbf{b}_0, \mathbf{b}_1\}$$

$$\mathbf{p}_0 \cdot (\mathbf{A} \cdot \mathbf{p}_0)$$

$$\mathbf{b}_0 (\mathbf{a}_0 \mathbf{b}_0 + \mathbf{a}_1 \mathbf{b}_1) + \mathbf{b}_1 (\mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1)$$

$$\mathbf{rsold} = \mathbf{r}_0 \cdot \mathbf{r}_0$$

$$\mathbf{b}_0^2 + \mathbf{b}_1^2$$

$$\mathbf{alpha}_0 = (\mathbf{r}_0 \cdot \mathbf{r}_0) / (\mathbf{p}_0 \cdot (\mathbf{A} \cdot \mathbf{p}_0))$$

$$\mathbf{b}_0^2 + \mathbf{b}_1^2$$

$$\mathbf{b}_0 (\mathbf{a}_0 \mathbf{b}_0 + \mathbf{a}_1 \mathbf{b}_1) + \mathbf{b}_1 (\mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1)$$

$$\mathbf{r}_1 = \mathbf{r}_0 - \mathbf{alpha}_0 * (\mathbf{A} \cdot \mathbf{p}_0)$$

$$\left\{ \mathbf{b}_0 - \frac{(\mathbf{a}_0 \mathbf{b}_0 + \mathbf{a}_1 \mathbf{b}_1) (\mathbf{b}_0^2 + \mathbf{b}_1^2)}{\mathbf{b}_0 (\mathbf{a}_0 \mathbf{b}_0 + \mathbf{a}_1 \mathbf{b}_1) + \mathbf{b}_1 (\mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1)}, \right. \\ \left. \mathbf{b}_1 - \frac{(\mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1) (\mathbf{b}_0^2 + \mathbf{b}_1^2)}{\mathbf{b}_0 (\mathbf{a}_0 \mathbf{b}_0 + \mathbf{a}_1 \mathbf{b}_1) + \mathbf{b}_1 (\mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1)} \right\}$$

/*sr₁ is short for “simplified r₁”, simplified result of r₁ stored in sr₁

* the following name rules is similar: “s”+“variable name” means the simplified variable.

*/

$$\mathbf{sr}_1 = \mathbf{Simplify}[\mathbf{r}_1]$$

$$\left\{ \frac{\mathbf{b}_1 ((-\mathbf{a}_0 + \mathbf{a}_2) \mathbf{b}_0 \mathbf{b}_1 + \mathbf{a}_1 (\mathbf{b}_0^2 - \mathbf{b}_1^2))}{\mathbf{a}_0 \mathbf{b}_0^2 + \mathbf{b}_1 (2 \mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1)}, \frac{\mathbf{b}_0 ((\mathbf{a}_0 - \mathbf{a}_2) \mathbf{b}_0 \mathbf{b}_1 + \mathbf{a}_1 (-\mathbf{b}_0^2 + \mathbf{b}_1^2))}{\mathbf{a}_0 \mathbf{b}_0^2 + \mathbf{b}_1 (2 \mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1)} \right\}$$

$$\mathbf{x}_1 = \mathbf{x}_0 + \mathbf{alpha}_0 * \mathbf{p}_0$$

$$\left\{ \frac{\mathbf{b}_0 (\mathbf{b}_0^2 + \mathbf{b}_1^2)}{\mathbf{b}_0 (\mathbf{a}_0 \mathbf{b}_0 + \mathbf{a}_1 \mathbf{b}_1) + \mathbf{b}_1 (\mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1)}, \frac{\mathbf{b}_1 (\mathbf{b}_0^2 + \mathbf{b}_1^2)}{\mathbf{b}_0 (\mathbf{a}_0 \mathbf{b}_0 + \mathbf{a}_1 \mathbf{b}_1) + \mathbf{b}_1 (\mathbf{a}_1 \mathbf{b}_0 + \mathbf{a}_2 \mathbf{b}_1)} \right\}$$

rsnew = sr₁ . sr₁

$$\frac{b_1^2 \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)^2}{\left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2} + \frac{b_0^2 \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right)^2}{\left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2}$$

srsnew = Simplify[rsnew]

$$\frac{(b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)^2}{\left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2}$$

beta = srsnew / rsold

$$\frac{\left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)^2}{\left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2}$$

p₁ = r₁ + beta * p₀

$$\left\{ b_0 - \frac{(a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2)}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)} + \frac{b_0 \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)^2}{\left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2}, \right. \\ \left. b_1 - \frac{(a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2)}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)} + \frac{b_1 \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)^2}{\left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2} \right\}$$

sp₁ = Simplify[p₁]

$$\left\{ \frac{(a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)}{\left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2}, \right. \\ \left. \frac{(a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2) \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right)}{\left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2} \right\}$$

In[73]:= **alpha₁ = (srsnew) / (sp₁ . (A.sp₁))**

$$\text{Out[73]} = \left((b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)^2 \right) / \left(\left(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1) \right)^2 \left(\frac{1}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} \right. \right. \\ \left. \left(a_1 b_0 + a_2 b_1 \right) (b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right) \left((a_0 (a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \right. \right. \\ \left. \left. \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right) \right) / (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2 + (a_1 (a_0 b_0 + a_1 b_1) \right. \right. \\ \left. \left. (b_0^2 + b_1^2) \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right) \right) / (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2 \right) + \\ \left. \frac{1}{(a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2} (a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2) \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right) \right. \\ \left. \left((a_1 (a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right) \right) / \right. \\ \left. (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2 + (a_2 (a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2) \right. \\ \left. \left. \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right) \right) / (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2 \right) \right) \right)$$

In[74]:= **salph₁ = Simplify[alpha₁]**

$$\text{Out[74]} = - \frac{a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)}{(a_1^2 - a_0 a_2) (b_0^2 + b_1^2)}$$

In[75]:= **r₂ = sr₁ - salph₁ * (A.sp₁)**

$$\text{Out[75]} = \left\{ \frac{b_1 \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right)}{a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)} + \right. \\ \left((a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)) \left((a_0 (a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right) \right) / \right. \\ \left. (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2 + (a_1 (a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2) \right. \\ \left. \left. \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right) \right) / (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2 \right) \right) / \\ \left((a_1^2 - a_0 a_2) (b_0^2 + b_1^2) \right), \frac{b_0 \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right)}{a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)} + \\ \left((a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1)) \left((a_1 (a_1 b_0 + a_2 b_1) (b_0^2 + b_1^2) \left((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2) \right) \right) / \right. \\ \left. (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2 + \right. \\ \left. (a_2 (a_0 b_0 + a_1 b_1) (b_0^2 + b_1^2) \left((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2) \right) \right) / \right. \\ \left. \left. (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))^2 \right) \right) / \left((a_1^2 - a_0 a_2) (b_0^2 + b_1^2) \right) \}$$

Factor[r₂]

{0, 0}

x₂ = x₁ + salpha₁ * sp₁

$$\left\{ \frac{b_0 (b_0^2 + b_1^2)}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)} - \frac{(a_1 b_0 + a_2 b_1) ((-a_0 + a_2) b_0 b_1 + a_1 (b_0^2 - b_1^2))}{(a_1^2 - a_0 a_2) (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))}, \right. \\ \left. \frac{b_1 (b_0^2 + b_1^2)}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)} - \frac{(a_0 b_0 + a_1 b_1) ((a_0 - a_2) b_0 b_1 + a_1 (-b_0^2 + b_1^2))}{(a_1^2 - a_0 a_2) (a_0 b_0^2 + b_1 (2 a_1 b_0 + a_2 b_1))} \right\}$$

sx₂ = Factor[x₂]

$$\left\{ \frac{a_2 b_0 - a_1 b_1}{-a_1^2 + a_0 a_2}, \frac{-a_1 b_0 + a_0 b_1}{-a_1^2 + a_0 a_2} \right\}$$

bncg = A.sx₂

$$\left\{ \frac{a_1 (-a_1 b_0 + a_0 b_1)}{-a_1^2 + a_0 a_2} + \frac{a_0 (a_2 b_0 - a_1 b_1)}{-a_1^2 + a_0 a_2}, \frac{a_2 (-a_1 b_0 + a_0 b_1)}{-a_1^2 + a_0 a_2} + \frac{a_1 (a_2 b_0 - a_1 b_1)}{-a_1^2 + a_0 a_2} \right\}$$

Simplify[bncg]{b₀, b₁}