Assumptions : (1 and 2 are based on Sylvester ' s criteria)

1. $a_0 > 0$
2. $a_0 a_2 - a_1^2 > 0$
3. $b_0 \neq 0 \;||\; b_1 \neq 0$

$\text{alpha}_0 = (r_0.r_0) / (p_0.(A.p_0))$

$$= \frac{b_0^2 + b_1^2}{b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1)}$$

Prove : $b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1) > 0$

$$= a_0 b_0^2 + 2 a_1 b_0 b_1 + a_2 b_1^2$$

$$= a_0 \left(b_0^2 + 2 \frac{a_1}{a_0} b_0 b_1 + \frac{a_2}{a_0} b_1^2\right)$$

$$= a_0 \left[\left(b_0 + \frac{a_1}{a_0} b_1\right)^2 - \frac{a_1^2}{a_0^2} b_1^2 + \frac{a_2}{a_0} b_1^2 \right]$$

$$= a_0 \left[\left(b_0 + \frac{a_1}{a_0} b_1\right)^2 + \frac{(a_0 a_2 - a_1^2)}{a_0^2} b_1^2 \right]$$

$$\frac{(a_0 a_2 - a_1^2)}{a_0^2} b_1^2 \geq 0 \text{ if } a_0 a_2 - a_1^2 > 0 \;\&\&\; b_0 \neq 0 \;||\; b_1 \neq 0$$

so, $\left[\left(b_0 + \frac{a_1}{a_0} b_1\right)^2 + \frac{(a_0 a_2 - a_1^2)}{a_0^2} b_1^2 \right] > 0$

and, $a_0 > 0$

$$a_0 \left[\left(b_0 + \frac{a_1}{a_0} b_1\right)^2 + \frac{(a_0 a_2 - a_1^2)}{a_0^2} b_1^2 \right] > 0$$

that is,

$b_0 (a_0 b_0 + a_1 b_1) + b_1 (a_1 b_0 + a_2 b_1) > 0$

DONE